There is no strong minimal pair

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joint work with
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What is an r.e. degree?

- A set A is *recursively enumerable (r.e.)* if $A = \text{dom } f$ for some partical recursive function f.
- $K = \{e \mid \Phi_e(e)\downarrow\}$ is an example of non-recursive *complete* r.e. set.
- 1944 Post's Problem: Is there a non-recursive incomplete r.e. degrees?
- 1957,1956 Friedberg-Muchnik Theorem: Yes (By a priority argument).
- 1964 Sacks' Density Theorem: Between any two comparable r.e. degrees, there is a third one. (By another priority argument)

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- Many people claimed it exists.

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(ii) $\varnothing <_T W$,

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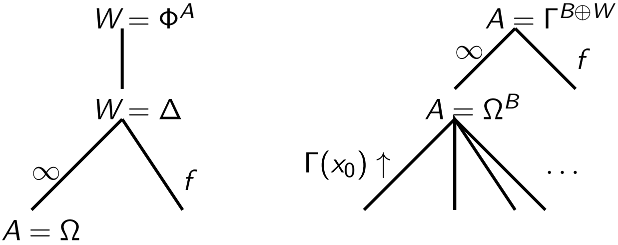
Requirements:

(i) $G_W : W = \Phi^A$,

(ii) $P_W(\Delta) : W \neq \Delta$ for all Δ,

(iii) $N_W(\Gamma) : A \neq \Gamma^{B \oplus W}$ for all Γ.

G_W, $P_W(\Delta)$ and $N_W(\Gamma)$



All together!