Permission within Ceteris Paribus

Huimin Dong

Zhejiang University

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Permission as Ideal Sufficiency

- Norms $\leftrightsquigarrow$ Ideality (e.g. moral standards, rightness, goodness, rational recommendations, solution concepts)
- Obligation: the necessary condition

$$R[w] \subseteq ||\varphi||$$

- For permission, two stories are involved:
  - Standard Deontic Logic: the dual of obligation [McNamara, 2014]
  - Strong Permission/Free Choice Permission (FCP): the sufficient condition [van Benthem, 1979, Dignum et al., 1996, Anglberger et al., 2015]

$$||\varphi|| \subseteq R[w]$$

    e.g.

    - "You may take an apple or take a pear."
    - "You may have a holiday tomorrow."
    - "You may vote 'High' in this game."

A Modal Logic for Deontic Necessity and Sufficiency

- Language $\{\neg, \wedge, \rightarrow, A, P, O\}$.
- Given a serial model $M = \langle W, R, ||\cdot||\rangle$ as a deontic model:

$$\overbrace{||\varphi|| \subseteq R[w]}^{P\varphi} \subseteq ||\varphi||$$
$$||\varphi|| \subseteq \underbrace{R[w] \subseteq ||\varphi||}_{O\varphi}$$

- Axiomatization [van Benthem, 1979]

A is a universal modality	$A\varphi \rightarrow O\varphi$
O is a D modality	$A\neg\varphi \rightarrow P\varphi$
$P\varphi \wedge P\psi \rightarrow P(\varphi \vee \psi)$	$O\varphi \wedge P\psi \rightarrow A(\psi \rightarrow \varphi)$
$\varphi \rightarrow \psi / P\psi \rightarrow P\varphi$	$\varphi/\triangle\varphi$, where $\triangle \in \{A, O\}$

- The FCP problem: the "master-slave" game [Lewis, 1979], the Hi-Lo game [Bacharach, 2006].

The “Master-Slave” Game

$\varphi \to \psi / P\psi \to P\varphi$

Your Master: It is permitted to have holiday tomorrow. [Lewis, 1979]

The Generic FCP

- FCP as normic laws [Pelletier and Asher, 1997]:

  "φ is permitted" iff "an instance of φ would be normatively okay."

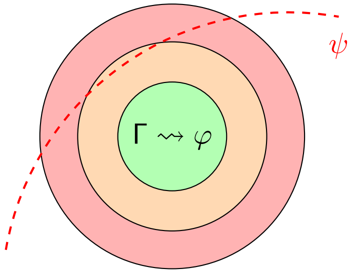
  1. It is intended to guide our *expectation* as to which actions will be good to execute normally.
     - E.g. "You may have a holiday tomorrow" [Lewis, 1979].
     - Illustrated as similarity/likelihood by using *plausibility* [Lewis, 1973].
  2. Normic laws are exception-tolerating.
     - In the absence of specific *reasons*, the normic laws will remain unchanged.
     - In other words, given a specific reason as *ceteris paribus*, the normaic laws might be changed, depending on how strong the reason is.
     - E.g. To revise the "master-slave" game: "Tomorrow is Christmas eve. You may have a holiday and drink the wine."

Our Proposal for Permission

The normal/most likely instances of φ is sufficient for ideality:

$$\max_{\leq_w}(||\varphi||) \subseteq R[w]$$

FCP and *Ceteris Paribus* Law



> Ceteris paribus, an increase of demand leads to an increase of prices.

Two approaches of *ceteris paribus* based on plausibility:

- Equality: Γ is used to select and update its equivalence class for CP [van Benthem et al., 2009, Grossi et al., 2015];
- Normality: Reprioritize regarding to Γ [Girard and Triplett, 2017].

The Static Part

- $M = \langle W, R, \{\leq_w\}_{w \in W}, ||\cdot||\rangle$ is a deontic model, with

$$\max_{\leq_w}(X) = \{v \in X \mid \forall u \in X \text{ s.t. } u \leq_w v\}$$

- Language $\{\neg, \wedge, \rightarrow, \trianglelefteq, \Box, P, O\}$
- Truth conditions:

$$\begin{array}{rcl} w \in ||\varphi \trianglelefteq \psi|| & \text{iff} & \forall u \exists v \text{ s.t. } (u \in ||\varphi|| \Rightarrow v \in ||\psi|| \;\&\; u \leq_w v) \\ w \in ||\Box(\varphi/\psi)|| & \text{iff} & \max_{\leq_w}(||\varphi||) \subseteq ||\psi|| \\ w \in ||P\varphi|| & \text{iff} & \max_{\leq_w}(||\varphi||) \subseteq R[w] \\ w \in ||O\varphi|| & \text{iff} & R[w] \subseteq ||\varphi|| \end{array}$$

- $A\varphi := (\neg\varphi) \trianglelefteq \bot$ and $E\varphi := \neg A \neg \varphi$
- $\Box\varphi := \Box(\top/\varphi)$ and $\Diamond\varphi := \neg\Box\neg\varphi$
- $\Box(\varphi \mid \psi) := \Box(\varphi/\psi) \wedge \Box(\psi/\varphi)$

Various Important Validities

- "Obligation as the weakest permission": $(O\varphi \wedge P\psi) \to \Box(\psi/\varphi)$
- Kant's "ought implies can": $O\varphi \to E\varphi$
- Free choice: $P\varphi \wedge P\psi \to P(\varphi \vee \psi)$
- Solution to the Lewis problem: $P\varphi \wedge \Box(\psi/\varphi) \to P\psi$
- Indifferent salience proposed by Kamp: $P(\varphi \vee \psi) \wedge \Box(\varphi \mid \psi) \to P\varphi \wedge P\psi$
- "Permission to fail": $P\bot$

Axiomatization for the Static Logic

Theorem

The system in below is sound and (weak) complete.

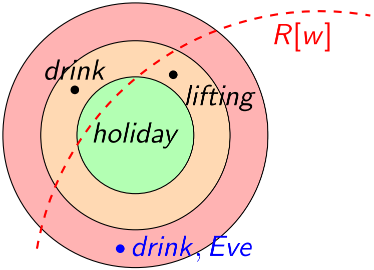
- Tautologies
- The binary modality $\trianglelefteq$ satisfies the axioms and rules suggested in [Halpern, 1997]
- The binary modality $\Box$ satisfies the axioms and rules suggested in [Burgess, 1981]
- O is a D-modality
- OiE: $O\varphi \to E\varphi$
- PtF: $P\bot$
- RFCP: $P\varphi \wedge P\psi \to P(\varphi \vee \psi)$
- FCP: $P\varphi \wedge \Box(\psi/\varphi) \to P\psi$
- OWP: $O\varphi \wedge P\psi \to \Box(\psi/\varphi)$

Solving the FCP: Part I

The FCP in the "Master-Slave" Game

Given "The Slave may have a holiday," what can we have:

1. "The Slave may have a holiday and drink the Master's wine." ✖
2. "The Slave may have a holiday and in the gym lifting weights." ✔
3. "Tomorrow is Christmas eve. The Slave may have a holiday and drink the Master's wine, but may not have a holiday and in the gym lifting weights." ?



- $\neg P(drink)$
- $P(lifting)$

Models for Specificity

Define a model $C^{\Gamma} = \langle C(\Gamma), \preceq \rangle$ to represent the specific instances w.r.t. the given context Γ:

- $C(\Gamma) = \{\{\pm p \mid p \text{ is an atomic proposition occurs in } \Gamma\} \mid$ $\text{either } \pm p = p \text{ or } \pm p = \neg p\}$;
- $\preceq \subseteq C \times C$ is reflexive, transitive, and connected.

Given $c \in C(\Gamma)$, we simplify $M, w \models \bigwedge_{\pm p \in c} \pm p$ as $M, w \models c$.

The updated model $M \otimes C^{\Gamma} = \langle W^*, R^*, \leq^*, V^* \rangle$ is defined as follows:

- $W^* = \{(u, c) \mid M, u \models c \text{ where } c \in C\}$; (Eliminative)
- $(u, c) \leq^*_w (v, d)$ iff either $c \prec d$ or $c \sim d$ but $u \leq_w v$; (Lexicographic)
- $(u, c) R^* (v, d)$ iff uRv and $c \preceq d$; (Eliminative)
- $(u, c) \in V^*(p)$ iff $u \in V(p)$.

$$M, w \models \langle \Gamma \rangle \varphi \text{ iff } \exists (w, c) \in W^* \text{ s.t. } M \otimes C^{\Gamma}, (w, c) \models \varphi$$

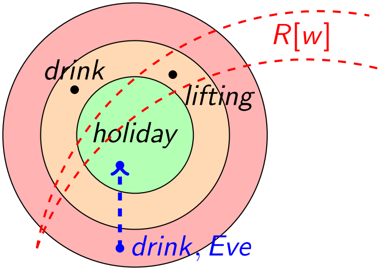
Solving the FCP: Part II

The FCP in the "Master-Slave" Game

Given "The Slave may have a holiday," what can we have:

1. "The Slave may have a holiday and drink the Master's wine." ⊗
2. "The Slave may have a holiday and in the gym lifting weights." ☑
3. "Tomorrow is Christmas eve. The Slave may have a holiday and drink the Master's wine, but may not have a holiday and in the gym lifting weights." ☑

- $\neg P(drink)$
- $P(lifting)$
- $[\{Eve\}](P(drink) \land \neg P(lifting))$



Theorem

The system in below is sound and (weak) complete.

- $[\Gamma]p \leftrightarrow \bigwedge_{c\in C}(c \to p)$
- $[\Gamma]\varphi \wedge \psi \leftrightarrow [\Gamma]\varphi \wedge [\Gamma]\psi$
- $[\Gamma]\neg\varphi \leftrightarrow \bigwedge_{c\in C}(c \to \neg[\Gamma]\varphi)$
- $[\Gamma]O\varphi \leftrightarrow \bigwedge_{c\in C}(c \to \bigwedge_{d\succeq c} O(d \wedge \langle\Gamma\rangle\varphi))$
- $[\Gamma]P\varphi \leftrightarrow \bigwedge_{c\in C}\{c \to \bigwedge_{d\in C}[(A\bigwedge_{e\succ d}\Gamma^e_\varphi \to P\bigvee_{e\sim d}\neg\Gamma^e_\varphi)\wedge \bigwedge_{d\preceq c}\Box(\bigvee_{e\sim d}\neg\Gamma^e_\varphi / E\neg\bigwedge_{e\succ d}\Gamma^e_\varphi)]\}$
- $[\Gamma](\varphi \trianglelefteq \psi) \leftrightarrow \bigwedge_{c\in C}\{c \to \bigwedge_{d\in C}[A(\bigvee_{e\sim d}\neg\Gamma^e_\varphi \to E\bigvee_{e\succ d}\neg\Gamma^e_\psi) \vee (\bigvee_{e\sim d}\neg\Gamma^e_\varphi) \trianglelefteq (\bigvee_{e\sim d}\neg\Gamma^e_\psi)]\}$

where $\Gamma^e_\varphi := e \to [\Gamma]\neg\varphi$.

Concluding Remarks

We have:

- present an entanglement between plausibility and ideality in natural language and games;
- a sound and (weak) complete dynamic logic for permission and reasons as *ceteris paribus*, with various important validities in deontic logics;
- a solution to the FCP, which can be extended to solve some other normative issues, e.g. in game theory;
- a comparison with a deontic logic for the thesis of “Good,” and then a defense of the ethical thesis of “Right.”

Future works:

- Objective Likelihood $\rightarrow$ Subjective Likelihood?
- Non-connected likelihood order?
- Obligation as the necessary condition of *normal* normative fineness?
- Game theory?

Thanks for your attention!

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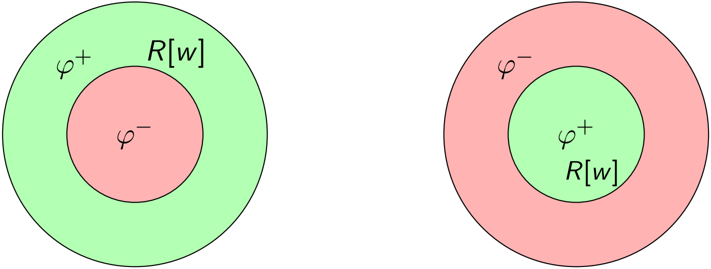
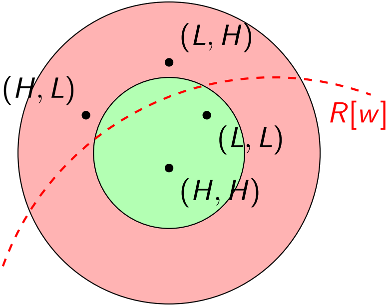
Reasons to Update Permissions



- $O\psi \wedge (\psi \to \varphi) \to [\uparrow \psi]P\varphi$, where $[\uparrow \psi]$ is an upgrade operator [van Benthem et al., 2014].

Refinement of the Hi-Lo: Part I

		Player A High	**Player A** Low
Player B	High	2, 2*	0, 0
Player B	Low	0, 0	2, 2*



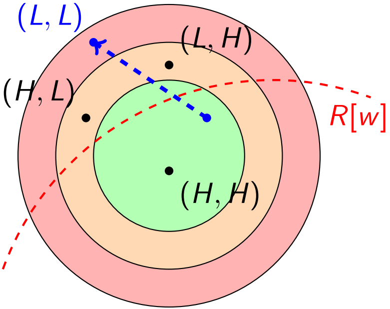
The FCP in the Hi-Lo Games

From an action-guidance point of view, can we say:

1. "*Given the choice 'High' of the other*, you may vote 'High'."
2. "*Given the choice 'Low' of the other*, you may vote 'Low'."

Refinement of the Hi-Lo: Part II

		Player A	
		High	Low
Player B	High	2, 2*	0, 0
	Low	0, 0	2, 2



The FCP in the Hi-Lo Games

From an action-guidance point of view, can we say:

1. "*Given the choice 'High' of the other*, you may vote 'High'."
2. "*Given the choice 'Low' of the other*, you may vote 'Low'."

Risk dominance: [Harsanyi and Selten, 1988]